Fear and Economic Behavior

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Abstract: Fear is an important factor in decision making under risk and uncertainty. Psychology research suggests that fear influences one's risk attitude and fear may have important consequences for decisions concerning for example investments, crime, conflicts, and politics. I model strategic interactions between players who can be either in a neutral or a fearful state of mind. A player's state of mind determines his or her utility function. The two main assumptions are that (i) fear is triggered by an increase in the probability or cost of negative outcomes and (ii) a player in the fearful state is more risk averse. A player's beliefs over the probability and cost of negative outcomes determine how the player transitions between the states of mind, and I use psychological game theory to analyse three applications. The applications illustrate how fear of a bank run can spread among bank clients and cause a bank panic, how a player can use knowledge of others' fear sensitivity to own advantage, and how fear can cause an apparent overreaction to bad news.

Keywords: emotions; fear; risk aversion; psychological game theory

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1 Introduction

Fear influences one's judgement, behavior, and decision making. A fearful person shows an increased concern with risk and tends to be less willing to participate in risky lotteries. For instance, a person who becomes fearful after an extreme stock market fall may overreact and sell more than he or she otherwise would have. Although fear is an important factor in decision making under risk and uncertainty, few economists have studied the topic and a formal analysis of the behavioral consequences of fear remains absent.[1]

This paper proposes a game theoretic model of players who can transition from a neutral to a fearful state of mind. A player's state of mind determines his or her utility function. While fear can manifest itself in many ways, in this paper fear is defined as an emotion with a specific trigger and which causes an increased concern with risk.[2] More specifically, the assumption is that a player is more risk averse in the fearful than in the neutral state of mind. By focusing on the behavioral consequences of fear as mediated through an increased risk aversion, I abstract from the negative utility from the experience of fear. I assume that a player transitions to the fearful state of mind after an increase in the probability or cost of negative outcomes. A negative outcome is an outcome bad enough to potentially instill fear when anticipated.

The players hold initial beliefs over the expected cost of negative outcomes when the game begins. A player transitions to the fearful state of mind after an increase in the expected cost of negative outcomes. Since the players' beliefs directly affect their utility functions, the game is a psychological game (Battigalli & Dufwenberg, 2009; Battigalli et al., 2019).[3] Consider for example a person who takes a walk in the park late at night. A negative outcome in this interaction

[1]The interest for fear and decision making has grown among empirical and experimental economists during the decade since the financial crisis (see e.g. Callen et al., 2014; Campos-Vazquez & Cuilty, 2014; Cohn et al., 2015; Dijk, 2017; Guerrero et al., 2012; Guiso et al., 2018; Kuhnen & Knutson, 2011; Malmendier & Nagel, 2011; Nguyen & Noussair, 2014; Wang & Young, 2020). The relationship between fear and risk attitudes is not found in all papers (see e.g. Alempaki et al., 2019; Gärtner et al., 2017).

[2]Psychology literature emphasize that once an individual becomes fearful, his or her concern with risk increases Holtgrave & Weber (see e.g. 1993); Lerner & Keltner (see e.g. 2000, 2001); Lerner et al. (see e.g. 2003); Loewenstein et al. (see e.g. 2001); Smith & Ellsworth (see e.g. 1985); Smith & Lazarus (see e.g. 1991).

[3]For a general discussion of psychological game theory and its usefulness in modeling emotions and other belief-dependent motivations, see e.g. Battigalli & Dufwenberg (2020).

is to be robbed of all of one's money and one may become fearful if a stranger approaches.

The role of fear is illustrated in three applications. The first application is a sequential game between a robber and a victim. This game illustrates how a player may use his or her knowledge of another player's fear sensitivity to bring about a desired outcome when the players' incentives are misaligned. The second is a simplified version of Diamond & Dybvig's (1983) seminal bank run game with the addition of an exogenous risk that a player 'needs money tomorrow' and is forced to withdraw. This game illustrates how fear can affect the outcome also when players' incentives are aligned, and how fear of a bank run can spread in a population and cause a bank run. The third application illustrates how fear may affect a player's willingness to take a vaccine when a public health authority informs him or her about a disease. This application highlights the tendency of fear to strengthen a player's response to information that increases the probability or cost of negative outcomes.

The observation that fear can be of importance in strategic interactions has been made before. Shelling (1980) discusses the strategic consequences of fear in a situation similar to the robbery game studied in this paper. Shelling considers a homeowner who investigates a noise at night with a gun in his hand, just to find a burglar, also armed with a gun. The situation has two equilibria. In one, no one shoots and the burglar leaves quietly. In the other, the homeowner and the burglar shoot each other. While neither of them prefers the shooting equilibria, Shelling notes that they may shoot not by calculation, but by nervousness. This situation can be formalized using the model proposed in this paper. If either the homeowner or the burglar were to become fearful, then shooting becomes the unique equilibrium.

Psychological game theory has been used to model other emotions, for example guilt and anger (Battigalli & Dufwenberg, 2007; Battigalli et al., 2019). Intuitively, anger is the emotion most closely related to fear. Battigalli et al. (2019) model players with a belief-dependent utility function that assigns a weight both to own and others' material payoff. In the absence of frustration, the weight on others' material payoff is zero. However, as a player's expected material payoff decreases, his or her frustration increases. As frustration increases, the player's negative concern for others' payoffs increases. Similar to the players studied in this paper, the players in (Battigalli et al., 2019) form initial beliefs over how the interaction will play out, and a disadvantageous change alters their utility function. However, Battigalli et al. study players who become frustrated if their expected material payoff decreases, whereas this paper studies players who transition to a fearful state of mind if their expected cost of negative outcomes increases. Moreover, while the triggers of frustration and fear are similar, frustration causes a player to have a negative concern for others' payoffs whereas fear causes an increased concerned with risk.

Caplin & Leahy (2001) propose a model of anticipatory emotions. They study a two-period model of lotteries with a mapping from physical lotteries to mental states. The decision maker's first-period utility may for example decrease in the variance of the second-period realizations of the lottery. Such a decision maker is said to experience anxiety prior to the resolution of the lottery and is less likely to take part in lotteries with high variance. By contrast, the focus of this paper is on the increased concern with risk in the fearful state of mind.

Another closely related paper is Kőszegi & Rabin (2007). Kőszegi & Rabin build on Kahneman & Tversky's (1979) work on prospect theory. They model a decision maker who evaluates an outcome relative to a reference point formed by the decision makers recent beliefs. The decision maker's utility is a combination of a reference-independent "consumption utility" and a "gain loss" utility that depends on the difference between the consumption utility and the reference point.

The decison maker's reference point determines how much risk he or she is willing to take on. The reference point can be either deterministic or stochastic. A decision maker who expects risk views a lottery as less aversive than a decision maker who does not expect risk to start with. Moreover, the decision maker is sophisticated in the sense that he or she correctly predicts the environment and own behavior in the environment.

Similarly, in this paper a player's transition to the fearful state of mind depends on the player's initial beliefs over how the interaction will play out. The players are also sophisticated in the sense that they can correctly predict own and others' state transitions and the behavioral consequences. However, while Kőszegi & Rabin's decision maker has a reference-dependent utility function, the players in my model have belief-dependent transition probabilities between their states of mind. Further, Kőszegi & Rabin's decision maker is concerned with expected consumption utility and any deviations therefrom, whereas the players in this paper are concerned with the expected cost of negative outcomes.

Dillenberger & Rozen (2015) also study decision makers whose risk attitude may change during the decision making process. They model a decision maker who makes repeated decisions over lotteries. The decision maker becomes more risk averse after a disappointing realization than after an elating. Realizations are classified as disappointing or elating using a threshold rule. By contrast, this paper studies players who may transition to a fearful state of mind when the expected cost of negative outcomes increases.

This paper proceeds as follows. Section 2 presents the model and preliminaries of psychological game theory. Sections 3, 4, and 5 apply the model to three interactions, a robbery game, a bank run game, and a public health intervention. Section 6 discusses the model and concludes.

2 The Model

2.1 Preliminaries

Game form The focus of this paper is on a class of finite multi-stage game forms with observed actions and perfect recall.[4] Let $I = \{1, ..., n\}$ denote the finite set of personal players. The game form may contain chance moves or moves by "nature" denoted by player 0. Let $I_0 = I \cup \{0\}$.

The multi-stage game consists of $L + 1$ stages indexed by $l \in \{0, ..., L\}$. At the end of each stage, all players observe the stage's action profile. Let $a^0 \equiv (a_0^0, a_1^0, ..., a_n^0)$ be the stage-0 action profile. At the beginning of stage 1, players know history h^1, which can be identified with a^0. Similarly, define h^{l+1}, the history at the end of stage l, to be the sequence of actions in the previous periods, $h^{l+1} = (a^0, a^1, ..., a^l)$. Let the initial, or empty, history be denoted by h^0, the set of non-terminal histories denoted by H, and let the set of terminal histories, or plays, h^{L+1}, be denoted by Z.

Let $A_i(h^l)$ denote the feasible actions of player $i \in I$ in stage l when the history is h^l, and let $A(h^l) = \times_{i \in I} A_i(h^l)$. In each stage the players, including

[4] A game form (or a game protocol) specifies the structure of a strategic situation: the players, how they can choose, and the material consequences of their actions.

chance, simultaneously choose actions from a finite subset of (potentially) history-dependent feasible actions $A_i(h^l)$. This can be done without loss of generality since an *inactive* player can be represented as a player whose feasible action $A_i(h^l)$ is a singleton with "do nothing" as the only action.

If chance is active at h^l, its move is specified by the probability mass function $p_0(\cdot|h) \in \Delta(A_0(h^l))$. Chance selects a feasible action at random, and the action is revealed to the players after the stage. The players have identical priors on the probability of chance's actions.

The *material* payoffs of the players' actions are determined by an outcome function $\pi : Z \to \mathbb{R}^n$ that associates each play z with a profile of material payoffs.

Beliefs Players form beliefs over own and others' behavior, and over others' beliefs about behavior. A player's beliefs are modeled as a hierarchical conditional probability system (Battigalli et al., 2019). A player's beliefs over own and other's behavior – the plays $z \in Z$ – are called *first-order beliefs*. They are defined for each history $h \in H$. The first-order beliefs are denoted by $\alpha_i(\cdot|Z(h)) \in \Delta(Z(h))$, where $\Delta(Z(h))$ is the set of probability measures on $Z(h)$. The system of beliefs $\alpha_i = (\alpha_i(\cdot|Z(h)))_{h\in H}$ must satisfy two properties. First, Bayes' rule for conditional probabilities must hold whenever defined. Second, if in stage l player i moves simultaneously with other players, then i must believe that the simultaneous actions of the co-players are statistically independent of i's action.[5]

The first-order beliefs α_i, are composed of two parts: player i's beliefs over own and over other's behavior. The beliefs over own behavior, $\alpha_{i,i} \in \times_{h\in H}\Delta(A_i(h))$, take the form a behavior strategy. They can be interpreted as the player's *plan* since they are the result of the player's contingent planning of which action to take at each history.[6]

A player's beliefs over other players' first-order beliefs constitute his or her *second-order beliefs*. Let $\Delta_{i,1}$ denote player i's space of first-order beliefs. Second-order beliefs are systems of conditional probabilities over both plays, $z \in Z$, and co-players' first-order beliefs, $\alpha_{-i} \in \times_{j\neq i}\Delta_{j,1}$, for each history $h \in H$. Player i's second-order beliefs are denoted by $\beta_i = (\beta_i(\cdot|h))_{h\in H} \in \times_{h\in H}\Delta(Z(h) \times_{j\neq i} \Delta_{j,1})$.

[5]See Battigalli et al. (2019) or Battigalli & Dufwenberg (2020) for a more detailed specification of the belief hierarchies and their properties.

[6]The beliefs over the behavior of others, $\alpha_{i,-i} \in \times_{h\in H}\Delta(A_{-i}(h))$, also constitute a behavior strategy if there is only one other player.

Player i's space of second-order beliefs are denoted by $\Delta_{i,2}$. It can be shown that the first-order beliefs α_i can be derived from the second-order beliefs β_i such that beliefs of different orders are mutually consistent (see e.g. Battigalli et al., 2019).

2.2 States of mind

This paper proposes a model of players who can transition from a neutral to a fearful state of mind. A player's preferences at a given history depend on material consequences and the player's state of mind. The players studied in this paper are sophisticated in the sense that they correctly anticipate own and others' transitions to the fearful state of mind, and how the fearful state affect's preferences. However, the model allows for the study of players who are either partially naïve, and mistaken about either transition probabilities or preferences, or naïve and mistaken about both.

As will be shown, transitioning to a fearful state of mind may cause a welfare loss. While defining welfare for emotional players is a non-trivial issue, this paper measures welfare as the preferences in the neutral state of mind.[7] In other words, a player's welfare corresponds to his or her material payoff.[8]

State transitions Psychology research suggests that emotions have a specific stimuli and a clear starting point and neuroscience research says that the purpose of the fear system is to detect warning signals for impending threats (see e.g. LeDoux, 1998). A wide variety of external stimuli may trigger a fear response.[9] Fear stimuli differ between cultures and individuals, and are modelled as exogenously defined for each player and game, and treated as primitives. The payoffs of each interaction are normalized such that only outcomes bad enough to, potentially, trigger a fear response when anticipated have a negative material payoff.[10]

[7]This is similar to the approach in Bernheim & Rangel (2004).

[8]Behavioral welfare economics is a field of its own (see e.g. Bernheim & Rangel, 2009, for a discussion).

[9]Some common fear stimuli are snakes, heights, auto accidents, being in a fight, and losing one's job (Geer, 1965).

[10]A consequence of this modelling choice is increased modeller freedom. As will be shown, the set of equilibria is sensitive to small changes in payoff around zero.

Player i's set of negative outcomes is defined as

$$Z_{i,-} := \{z \in Z : \pi_i(z) < 0\}. \tag{1}$$

The emotional system is more likely to respond to changes than to levels of stimuli (Frederick & Loewenstein, 1999) and each player continuously assesses his or her situation to detect warning signals.

I define player i's *peril* at history h, given his or her first-order belief system α_i as:

$$P_i(h|\alpha_i) = \sum_{z \in Z_{i,-}} \alpha_i(z|h)|\pi_i(z)|, \tag{2}$$

where $\alpha_i(z|h)$ is player i's belief, conditional on history h, in the negative outcome $z \in Z_{i,-}$, and $|\pi_i(z)|$ is the cost of the outcome. In other words, a player's peril is his or her expected cost of negative outcomes at history h. Peril increases in both the probability and cost of negative outcomes.

A warning signal for player i at history h, given the first-order belief system α_i is defined as an increase in peril compared with i's initial peril:

$$P_i'(h|\alpha_i) = \max\{0, P_i(h|\alpha_i) - P_i(h^0|\alpha_i)\}. \tag{3}$$

The assumption is that the interactions are fast enough for the initial peril to be the relevant reference point. Moreover, once in the fearful state of mind, the players cannot return to the neutral. Research has shown that emotions tend to linger after the source of the emotion has vanished (Andrade & Ariely, 2019). Since the interactions are fast, there is not enough time for the players to calm down even if peril decreases.

The increase in peril required to transition to the fearful state of mind may differ between players. This heterogeneity is modeled with a sensitivity parameter τ_i, which is interpreted as player i's fear threshold.[11]

Player i transitions from the neutral to the fearful state of mind if

$$P_i'(h|\alpha_i) \geq \tau_i. \tag{4}$$

In other words, player i transitions when the increase in peril is more than he or

[11] A player's fear threshold, or fear sensitivity, can be thought of as an innate personality trait.

she can bare.

Preferences For simplicity of analysis, preferences in the two states of mind correspond to the extreme cases of risk neutrality in the neutral state of mind and a maximal concern with risk in the fearful. In the neutral state of mind, the player maximizes own expected material payoff. In the fearful state of mind, the player experiences highly intense fear such that his or her coefficient of risk aversion goes to infinity, and the player views a gamble in terms of its worst-case scenarios. A fearful player's utility function corresponds to the maximin utility function.

Psychology research has long emphasized the relationship between fear and risk attitudes. Modern psychology research uses the appraisal-tendency framework (Lerner & Keltner, 2000) to study emotions. This framework states that each emotion gives rise to a cognitive predisposition to appraise future events in line with one or more appraisal themes.[12] There are five central appraisal themes: certainty, pleasantness, attentional activity, anticipated action, and control. Fear is associated with a sense of uncertainty and a lack of a sense of control, both of which are factors that influence judgments of risk.[13]

The appraisal-tendency framework predicts that fear causes a person to be less willing to take risks. The effect of fear on risk attitude has been found to have two main mechanisms (see e.g. Cohn et al., 2015; Guiso et al., 2018; Lerner & Keltner, 2000, 2001; Lerner et al., 2003; Wang & Young, 2020). First, a fearful person tends to behave as if their risk aversion has increased. Second, the subjective probabilities the fearful person assigns to dangerous outcomes increases. The focus of this paper is on situations of risk rather than uncertainty and I model the effect of fear on behavior as increased risk aversion.

Decision utility The players' "decision utility" functions are defined using the above formalization of fear. A player i moving at history h maximizes a belief-dependent decision utility function $u_i : A_i(h) \times \Delta_{i,2} \to \mathbb{R}$ for $i \in I$, $h \in H$, defined

[12] Those familiar with Frijda's (1986) action-tendency framework for emotions might notice that the appraisal-tendency framework can be viewed as an extension of this earlier framework.

[13] Fear is often associated with unpleasantness. I restrict the focus of this paper to the change in the utility function rather than the utility from experiencing fear.

by:

$$u_i(h, a_i; \beta_i) = \begin{cases} \min_{a_{-i} \in A_{-i}(h)} E[\pi_i|(h, a_i, a_{-i})] & \text{if } P_i'(h; \alpha_i) \geq \tau_i \\ \mathbb{E}[\pi_i|(h, a_i); \beta_i] & \text{otherwise} , \end{cases} \tag{5}$$

where α_i is derived from β_i; $P_i'(h; \alpha_i)$ is the increase in peril at history h given beliefs α_i; and τ_i is i's fear threshold.

While fear is solely determined by the player's first-order beliefs, a player intending to use others' fear to his or her own advantage forms second-order beliefs over co-players' first-order beliefs.

Note that while each decision utility function is belief-independent, the transition between the states of mind is belief-dependent. In addition, because the players begin in the neutral state of mind and fear is triggered by an increase in peril, the decision utility at the root coincides with expected material payoffs. Fear is possible at end nodes, but cannot influence subsequent choices as the game is over and this paper abstracts from the disutility fear may cause.

Remark Players who can transition between states of mind typically behave as if they have time-inconsistent preferences. While their preferences are time-consistent within each state of mind, a player may prefer one action in the neutral state and another in the fearful. This may lead to self-control problems and a player in the neutral state of mind may be willing to invest in a commitment device to constrain the actions of a future fearful self.

In applications of present-biased preferences it is typical to consider a decision maker as consisting of multiple selves, one for each time period (O'Donoghue & Rabin, 2001; Thaler & Shefrin, 1981). In a similar way it is possible to consider a player who can transition between states of mind as a sequence of multiple selves, one for each state of mind.

2.3 Solution Concept

Because the transition to the fearful state of mind is belief-dependent, the games are psychological games in the sense of Battigalli & Dufwenberg (2009). The solution concept I use is the sequential equilibrium (SE) for psychological games (Battigalli & Dufwenberg, 2009; Battigalli et al., 2019). The SE for psychological games is an extension of Kreps and Wilson's (1982) classical notion of a sequential equilibrium. The games analyzed are one-shot interactions, and the equilibria are interpreted as the commonly understood ways to play the game by rational agents.

I consider games of complete information where the rules of the game and the players' fear thresholds and state dependent preferences are common knowledge.[14] An SE is an assessment: a profile of behavior strategies σ_i – the player's plans – and conditional second-order beliefs β_i such that σ_i is the plan $\alpha_{i,i}$ derived from the second-order belief β_i. While the SE concept gives equilibrium conditions for infinite belief hierarchies, the applications of this paper only depend on first- and second-order beliefs and the SE is defined up to second-order beliefs.

Definition 1 (Battigalli, Dufwenberg & Smith, 2019)**.** Assessment $(\sigma_i, \beta_i)_{i\in I}$ is *consistent* if for all $i \in I$, $h \in H$, $a = (a_j)_{j\in I} \in A(h)$
1. $\alpha_i(a|h) = \Pi_{j\in I}\sigma_j(a_j|h)$,
2. $marg_{\Delta_{-i,1}}\beta_i(\cdot|h) = \delta\alpha_{-i}$;

here α_i is derived from β_i and $\delta\alpha_{-i}$ is the Dirac measure assigning probability 1 to $\{\alpha_{-i}\} \subseteq \Delta^1_{-i}$.

The first condition requires players' beliefs about actions to satisfy independence across co-players, and after a deviation of player j player i expects j to behave in the continuation game as specified by j's plan $\alpha_{j,j}$. Thus, all players have the same first-order beliefs.

The second condition requires players' beliefs about co-players' plans to be correct and never change, on or off the path. Any two players thus share the same initial first-order beliefs about any other player and every player is able to infer the correct beliefs of his or her co-players.

Definition 2 (Battigalli, Dufwenberg & Smith, 2019)**.** Assessment (σ_i, β_i) is a *sequential equilibrium* if it is consistent and satisfies the following sequential rationality condition:
for all $h \in H$ and $i \in I(h)$, $\sup \sigma_i(\cdot|h) \subseteq \underset{a_i\in A_i(h)}{\arg\max} u_i(h, a_i|\beta_i)$.

[14] This is not a limitation to the model since it can be extended to games of incomplete information. For an analysis of a psychological game of incomplete information see Attanasi et al. (2016).

This definition coincides with the traditional definition of sequential rationality when players have standard preferences. An SE always exists when the utility functions are continuous (Battigalli et al., 2019). The utility function analyzed in this paper is discontinuous around τ_i, the fear threshold. A consequence is that a SE does not always exist. The situation is illustrated in the first application. However, if τ_i is sufficiently large, such that it is greater than the maximum increase in peril and the players cannot transition to the fearful state of mind, then there always is an SE that coincides with the SE in the game between players with standard preferences.

3 Robbery

A walk in the park Consider the example of a person who takes a walk in the park late at night, running the risk of being robbed. This interaction can be modeled as the game in Figure 1. There are two players, the robber (player 1) and the victim (player 2). The game has three stages. Player 1 is active in stage 0 and stage 2 (player 2's only action in these stages is "do nothing"). Only player 2 is active in stage 1 (player 1's only action is "do nothing"). The game has no chance moves and any risk is endogenous.

Player 1 first decides whether to attempt a robbery (a) or not (n). Player 2 decides, conditional on an attempt, whether to comply (c) or resist (r). Player 1 decides, conditional on player 2 resisting, whether to flee the scene (f) or use violence (v) to force the robbery.

The payoffs are normalized such that all outcomes following a robbery attempt have a negative payoff for player 2. If player 1 does not attempt a robbery, both players receive a zero payoff. If player 1 attempts a robbery and player 2 complies, then player 1 receives a payoff of 50, and player 2 a payoff of -50. If player 2 resists and player 1 flees, each receives a payoff of -10. Both players are better off without an attempt; player 1 avoids an unpleasant experience, and player 2 is not chased by the police. If player 2 resists and player 1 uses violence, then player 1 receives a payoff of -200 and player 2 a payoff of -500; player 2 is injured and player 1 is chased more fiercely by the police.

Clearly, the game between players with standard preferences has a unique SE in $((n, f), r)$. However, if player 2 is fearful at his or her decision node, then he

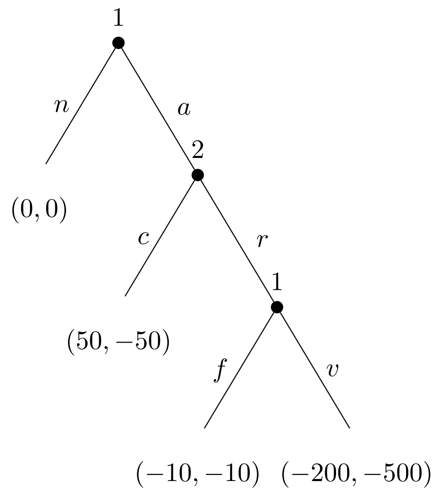

Figure 1: Robbery game.

or she maximizes the minimum payoff by choosing c. Player 1 has an incentive to ensure that player 2 transitions to the fearful state of mind if his or her decision node is reached. As we will see, player 1 can do so by randomizing between n and a.

Player 2's first-order beliefs α_2 can be split into beliefs over own plan, $\alpha_{2,2}$, and over player 1's plan, $\alpha_{2,1}$. Player 2's initial peril is

$$P_2\left(h^0|\alpha_2\right) = \left[50\left(1-\alpha_{2,2}^r\right) + \left(500\left(1-\alpha_{2,1}^f\right) + 10\alpha_{2,1}^f\right)\alpha_{2,2}^r\right]\alpha_{2,1}^a, \qquad (6)$$

where $\alpha_{2,1}^a$ is player 2's belief that player 1 plans a, $\alpha_{2,2}^r$ is player 2's plan of choosing r; and $\alpha_{2,1}^f$ is player 2's belief that player 1 plans f, conditional on r.[15]

Player 2's initial peril is zero if player 2 believes that player 1 plans n. All negative outcomes follows a, and the expected cost of negative outcomes depend on player 2's beliefs over the three terminal histories (a, c), $((a, f), r)$, and $((a, v), r)$. Initial peril is strictly increasing in player 2's belief that player 1 plans a. Initial peril also depends on player 2's own plan of choosing c or r, conditional on a.

Player 2's updated peril if his or her decision node is reached is

$$P_2\left(a|\alpha_2\right) = 50\left(1-\alpha_{2,2}^r\right) + \left(500\left(1-\alpha_{2,1}^f\right) + 10\alpha_{2,1}^f\right)\alpha_{2,2}^r. \qquad (7)$$

[15] This is a slight abuse of notation, but the abbreviations used to denote the conditional probabilities of actions derived from players' plans are justified by the increased readability of the analysis.

Since all negative outcomes follows a and all outcomes following a are negative, player 2's updated peril coincides with his or her expected material payoff.

Player 2's increase in peril is

$$P_2'(a|\alpha_2) = \left(1 - \alpha_{2,1}^a\right) \times \left[50\left(1 - \alpha_{2,2}^r\right) + \left(500\left(1 - \alpha_{2,1}^f\right) + 10\alpha_{2,1}^f\right)\alpha_{2,2}^r\right]. \tag{8}$$

The first factor is player 2's belief that player 1 will not choose a. The less probable player 2 believes a is, the larger the increase in peril if a occurs. If player 2 is certain that a will occur, $\alpha_{2,1}^a = 1$, then there is no increase in peril. The occurrence of a has already been taken into account. If player 2 is almost sure that a will not occur, $\alpha_{2,1}^a \approx 0$, then the increase in peril is approximately the expected cost of the negative outcomes that follows a. In other words, the smaller the $\alpha_{2,1}^a$, the greater the increase in peril. Note that the increase in peril also depends on player 2's own plan.

Player 1's knows player 2's fear threshold τ_2 and his or her optimal plan is to maximize the probability of a, conditional on player 2 transitioning to the fearful state of mind if the second decision node is reached. If the third decision node is reached, then player 1 maximizes own expected material payoff by choosing f.

Player 2's optimal plan depends on whether he or she transitions to the fearful state of mind after a. In the neutral state player 2 prefers r, and in the fearful state he or she prefers c. If τ_2 is sufficiently large, such that it is above player 2's maximum increase in peril after a, then player 2 cannot transition to the fearful state of mind regardless of own and player 1's plan. If τ_2 is sufficiently small, then player 1 can randomize between n and a such that player 2 transitions to the fearful state regardless of own plan. However, for intermediate values of τ_2, whether player 2 transitions to the fearful state of mind depends on his or her own plan.

Equilibria When $\tau_2 > 50$, the game has a unique SE in $((n, f), r)$, just as in a standard game. To check this, note that when $\tau_2 > 50$, then player 2 remains in the neutral state of mind regardless of own and player 1's plan. If his or her decision node is reached, then player 2 maximizes own material payoff by choosing r. Player 1 knows this and maximizes own expected payoff by choosing (n, f). Further, $((n, f), r)$ remains an SE also when $10 < \tau_2 \leq 50$. For this intermediate range of τ_2, player 2's state of mind depends on his or her own plan. If player 2 plans r, then he or she remains in the neutral state after a and maximizes own material payoff by choosing r. Player 1 maximizes own expected payoff by choosing (n, f). However, when $10 < \tau_2 \leq 50$, there is an additional SE in $\left(\left(\left[\frac{\tau_2}{50}, 1 - \frac{\tau_2}{50}\right], f\right), c\right)$. If player 2 plans c, then he or she transitions to the fearful state of mind after a, conditional on $P_2'(a|\alpha_2) = 50(1 - \alpha_{2,1}^a) \geq 10$. Player 1 can thus randomize such that player 2 transitions to the fearful state. Player 1 maximizes own expected payoff by choosing $\left(\left[1 - \frac{\tau_2}{50}, \frac{\tau_2}{50}\right], f\right)$. Player 2 transitions to the fearful state after a, and maximizes own minimum payoff by choosing c. Finally, when $\tau_2 \leq 10$, then $((n, f), r)$ cannot be an SE. To verify, assume it were. Player 2 plans r, and transitions to the fearful state if $P_2'(a|\alpha_2) = 10(1 - \alpha_{2,1}^a) \geq \tau_2$. Player 1 can randomize between n and a such that player 2 transitions to the fearful state by choosing $\alpha_{1,1}^a \leq 1 - \tau_2/10$. Player 2 transitions to the fearful state after a, and chooses c to maximize own minimum payoff, contradicting that $((n, f)r)$ is an SE. When $\tau_2 \leq 10$, the unique SE is $\left(\left(\left[\frac{\tau_2}{50}, 1 - \frac{\tau_2}{50}\right], f\right), c\right)$. If player 2 plans c, then he or she transitions to the fearful state if $P_2'(a|\alpha_2) = 50(1 - \alpha_{2,1}^a) \geq \tau_2$. Player 1 maximizes payoff by choosing $\left(\left[\frac{\tau_2}{50}, 1 - \frac{\tau_2}{50}\right], f\right)$. Player 2 transitions to the fearful state if his or her decision node is reached, and maximizes own minimum payoff by choosing c. □

In other words, if τ_2 is sufficiently large such that player 2 cannot transition to the fearful state of mind regardless of own plan, then the unique SE is identical to the SE in the game between players with standard preferences.

For intermediate fear thresholds, the game has two SE. If player 2 believes that he or she will not transition to the fearful state after a and plans r, then he or she will not transition and prefers r. If player 2 believes that he or she will transition to the fearful state and plans c, then he or she will transition and prefers c, conditional on player 1's randomization. Due to the own-plan dependency of peril, player 2's beliefs are self-fulfilling.[16]

Finally, if τ_2 is sufficiently small such that player 2 can transition to the fearful state of mind regardless of own plan, then the game has a unique SE. Player 1 maximizes a, conditional on player 2 transitioning should his or her decision node

[16]While games of complete and perfect information with no relevant ties always have a unique SE in standard games; this multiplicity of SE is not uncommon in psychological games and is due to own-plan dependency of the utility function.

be reached, and player 2 chooses c.

Proposition 1. If τ_2 is sufficiently small such that player 2 can transition to the fearful state of mind after a, then an SE exists in which $\alpha^a_{1,1} > 0$ and $\alpha^r_{2,2} = 0$. If τ_2 is sufficiently small such that player 2 can transition to the fearful state of mind after a regardless of own plan, then this SE is unique.

In this game, a low fear threshold implies a welfare loss for player 2 as measured in material payoffs. A player 2 who is in the fearful state of mind complies with the robbery attempt for a material payoff of -50, whereas a player 2 in the neutral state resists for a material payoff of -10. Moreover, since the fear thresholds are common knowledge, player 1 only makes an attempt with positive probability if τ_2 is sufficiently low. The lower τ_2, the higher the probability of an attempt.

Player 2's preferences are time inconsistent. In the neutral state of mind he or she would prefer to commit to r if such a commitment was possible. Player 2 can be interpreted as a player with two selves, one for each state of mind. Player 2 is in the neutral state of mind when the game begins. At his or her decision node, player 2 is either in the neutral or in the fearful state of mind depending on own and player 1's plan. The neutral self prefers to resist any attempt since player 1 would then flee the scene. However, the fearful self prefers to comply, and player 2's neutral self cannot control the fearful self.

The assumption that τ_2 is common knowledge is crucial for the analysis above. The caveat is that in reality, a robber finds it difficult to distinguish between fear sensitive and insensitive victims.

When τ_i is private information The robbery game can be extended to a situation in which τ_2 is private knowledge.

Assume player 1 meets a player 2 who is randomly drawn from a population of potential player 2's with uniformly distributed fear thresholds, $\tau_2 \sim U(0, 60)$. In other words, 1/6 are highly fear sensitive such that player 1 can always randomize such that they transition to the fearful state of mind should their decision node be reached. Another 1/6 are highly fear insensitive such that they cannot transition to the fearful state of mind regardless of own and player 1's plan. The remaining 4/6 may transition depending on their own plan. Assume, for simplicity, that half of them plans c and the other half r. Assume player 1 knows the population distribution of τ_2 and has to decide whether to make a robbery attempt.

Player 1 faces a trade of between increasing the probability of a successful robbery attempt, by decreasing the probability of choosing a, and increasing the probability of an attempt.

Player 1 chooses the probability of a that maximizes his or her expected material payoff

$$\max_{\alpha_{1,1}^a \in [0,1]} \left[50(\frac{6 - 5\alpha_{2,1}^a}{12}) - 10(1 - \frac{6 - 5\alpha_{2,1}^a}{12}) \right] \alpha_{1,1}^a. \tag{9}$$

The first term is the share of player 2's that will transition to the fearful state of mind and choose c, given their beliefs $\alpha_{2,1}^a$ of player 1 choosing a. In other words, it is the share of player 2's with $\tau_2 \leq 50(1 - \alpha_{2,1}^a)$, minus the share of player 2's with $\tau_2 \in [10, 50)$ whose self-fulfilling beliefs lead them to prefer r over c. The second term is the share of player 2's who remains in the neutral state of mind and chooses r. Conditional on r, player 1 chooses f, and receives a payoff of -10.

Player 1 maximizes his or her expected payoff by choosing $\alpha_{1,1}^a = \frac{2}{5}$. A player 2 with $\tau_2 \leq 10$ has a unique optimal plan in choosing c. If $\tau_2 \in (10, 30]$, then player 2 has self-fulfilling beliefs and both c and r are optimal plans. Finally, a player 2 with $\tau_2 \geq 30$ has a unique optimal plan in choosing r. The probability of a successful attempt is $1/3$.

Staying at home The game discussed above illustrates the intuition of the model, but it does not capture the full story of the person who takes a walk late at night. More realistically, player 2 makes an initial decision of whether to go for a walk (g) or stay at home (s). The corresponding game is illustrated in Figure 2. Note that player 2 now makes the first decision.

The game has four stages. Player 2 is active in stage 0 and 2, and player 1 in stage 1 and 3. Player 2 receives a material payoff of 5 from choosing g, conditional on n. If player 2 chooses s, then both players receive a zero payoff zero. Remaining payoffs are as in the previous example.

The game between players with standard preferences has a unique SE in $((g, r), (n, f))$. As before, the SE of the game between players who can transition to the fearful state of mind depends on τ_2.

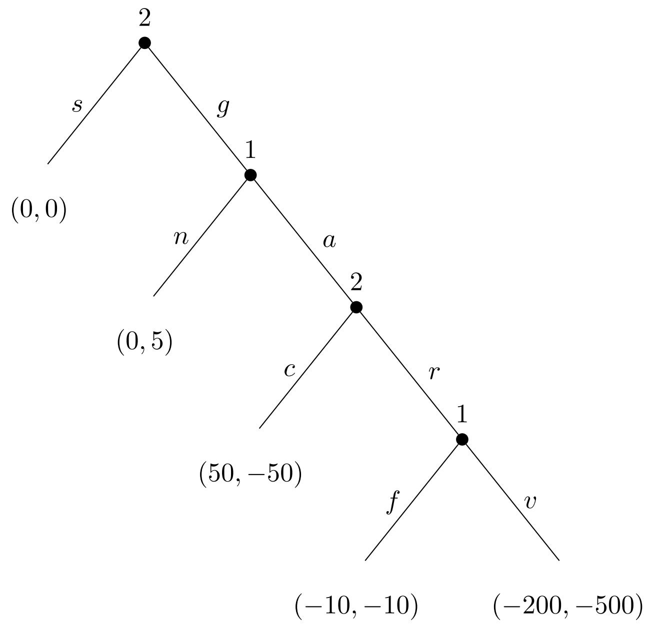

Figure 2: Extended robbery game.

Player 2's initial peril is

$$P_2\left(h^0|\alpha_2\right) = \left[50\left(1-\alpha^r_{2,2}\right) + \left(500\left(1-\alpha^f_{2,1}\right) + 10\alpha^f_{2,1}\right)\alpha^r_{2,2}\right]\alpha^g_{2,2}\alpha^a_{2,1}, \tag{10}$$

where $\alpha^g_{2,2}$ denotes player 2's plan of g. As before, player 2's peril depends on his or her own plan. Initial peril is zero if player 2 plans s or if player 2 believes that player 1 plans to choose n.

Player 2's updated peril if his or her second decision node is reached is

$$P_2\left((g,a)|\alpha_2\right) = 50\left(1-\alpha^r_{2,2}\right) + \left(500\left(1-\alpha^f_{2,1}\right) + 10\alpha^f_{2,1}\right)\alpha^r_{2,2}. \tag{11}$$

Player 2's updated peril, conditional on (g, a) corresponds to his or her expected material payoff.

The increase in peril is

$$P_2'\left((g,a)|\alpha_2\right) = \left(1-\alpha^g_{2,2}\alpha^a_{2,1}\right)\times \left[50\left(1-\alpha^r_{2,2}\right) + \left(500\left(1-\alpha^f_{2,1}\right) + 10\alpha^f_{2,1}\right)\alpha^r_{2,2}\right]. \tag{12}$$

If player 2 plans s with some positive probability, then the increase in peril is higher than in the previous example, should his or her decision node be reached.

Player 1's optimal plan is still to maximize the probability of an attempt, conditional on it being sufficiently small such that player 2 transitions to the fearful state of mind after (g, a). Note that if player 2 plans s with some positive probability, then player 1 may, depending on player 2's fear threshold, plan a with certainty. As before, player 1 plans f if player 2 chooses r.

Equilibria When $\tau_2 > 50$, the game has a unique SE in $((n, f), (g, r))$, just as in the standard game. Player 2 cannot transition to the fearful state of mind regardless of own and player 1's plan. As before, $((n, f), (g, r))$ remains an SE also when $10 < \tau_2 \leq 50$. For this intermediate range of τ_2, player 2's state of mind after (g, a) depends on his or her own plan. If player 2 plans (g, r), then he or she remains in the neutral state after (g, a), and maximizes own material payoff by choosing r. Player 1 maximizes own expected payoff by choosing (n, f). However, when $10 < \tau_2 \leq 50$, then $((a, f), (s, c))$ qualifies as another SE. If player 2 plans (s, c), then he or she transitions to the fearful state of mind after (g, a) if $P_2'((s, a)|\alpha_2) = 50 \geq \tau_2$. Thus, player 1 can guarantee a transition by planning a, and player 2 maximizes own minimum payoff by choosing c. Moreover, when $500/11 \leq \tau_2 \leq 50$, then there is an additional SE in $\left((g, c), \left(\left(\frac{\tau_2}{50}, 1 - \frac{\tau_2}{50}\right), f\right)\right)$. Player 2 plans (g, c) and transitions to the fearful state after (g, a) if $P_2'((g, a)|\alpha_2) = 50(1 - \alpha_{2,1}^a) \geq 10$. Player 1 can randomizes such that player 2 transitions by choosing $\alpha_{1,1}^a = 1 - \tau_2/50$. Player 2 maximizes own minimum payoff after (g, a) by choosing c. In addition, player 2 is in the neutral state of mind at his or her first decision node and maximizes own expected material payoff by choosing g since $\alpha_{2,1}^a = 1 - \tau_2/50$ is sufficiently small. Finally, when $\tau_2 \leq 10$, then $((n, f), (g, r))$ cannot be an SE. To verify, assume it were. Player 2 plans (g, r), and transitions to the fearful state if $P_2'((g, a)|\alpha_2) = 10(1 - \alpha_{2,1}^a) \geq \tau_2$. Player 1 can randomize between n and a such that player 2 transitions to the fearful state by choosing $\alpha_{1,1}^a \leq 1 - \tau_2/10$. Player 2 transitions to the fearful state after (g, a), and chooses c to maximize own minimum payoff, contradicting that $((n, f), (g, r))$ is an SE. When $\tau_2 \leq 10$ then $((a, f), (s, c))$ is the unique SE. If player 2 plans (s, c), then he or she transitions to the fearful state if $P_2'((s, a)|\alpha_2) = 50 \geq \tau_2$. Thus, player 1 can guarantee a transition by planning a, and player 2 maximizes own minimum payoff by choosing c. □

In other words, if τ_2 is sufficiently large such that player 2 cannot transition to the fearful state of mind regardless of own and player 1's plan, then the unique SE is identical to the SE in the game between players with standard preferences. For an intermediate range of τ_2, there is a multiplicity of SE. Because player 2's peril is own-plan dependent, his or her beliefs are self-fulfilling, and both $((n, f), (g, r))$ and $((a, f), (s, c))$ are SE. In addition, for a segment of the intermediate range there is a third SE in which player 2 plans (g, c). Since the probability of a is sufficiently small, player 2 maximizes own material payoff by choosing g. Player 2 is aware of the probability of a and that he or she will transition to the fearful state of mind and choose c conditional on a. Finally, if τ_2 is sufficiently small, such that player 2 may transition to the fearful state of mind regardless of own plan, then the unique SE is $((a, f), (s, c))$.

Proposition 2. If τ_2 is sufficiently small such that player 2 can transition to the fearful state of mind after (g, a), then there is an SE in which $\alpha_{1,1}^{a} > 0$ and $\alpha_{2,2} = (s, c)$. Moreover, if τ_2 is sufficiently small such that player 2 can transition to the fearful state after (g, a) regardless of own plan, then the SE is unique.

Fear insensitive victims go for a walk whereas fear sensitive victims stay inside and forego the utility of taking a walk. However, if the probability of a robbery attempt is sufficiently small, some fear sensitive victims may go for a walk while planning to comply if an attempt occurs.

As before, player 2 can be thought of as having two selves, one for each state of mind. At his or her first decision node, player 2 is the neutral self who prefers to go for a walk and resist any robbery attempt. However, the neutral self is aware that the fearful self may be in control at his or her second decision node. If τ_2 is sufficiently small, then the neutral self correctly anticipates that the fearful self will be in control at the second decision node. Depending on the probability of an attempt, the neutral self may either stay at home or go for a walk.

Observations The analysis of the robbery game relies crucially on player 2's payoffs following a robbery attempt being negative. More specifically, both the action preferred in the neutral state of mind, r, and the action preferred in the fearful state, c, lead to a negative outcome. Player 2 can transition to the fearful state of mind while planning c, and c is the optimal plan if player 2 transitions to the fearful state of mind.

The analysis changes if the maximin action leads to a non-negative outcome. Consider the game in Figure 3. Player 1 first decides whether to make an attempt. Conditional on an attempt, player 2 decides between c and r. If player 2 chooses r, then player 0 (chance) chooses v with probability ε. This can be interpreted as player 1 trembling, and, by accident, choosing v.

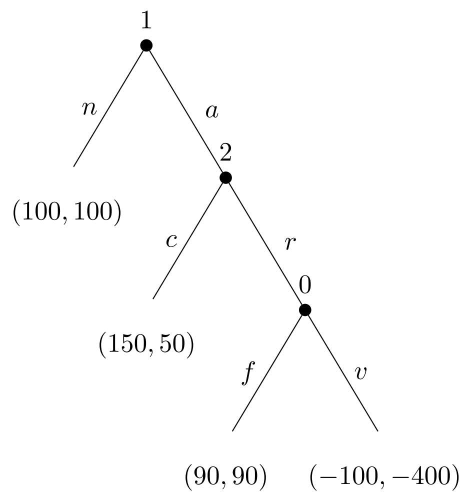

Figure 3: Robbery game with a single negative outcome.

In this game, a player 2 with a sufficiently small fear threshold cannot have an optimal plan. To see this, assume that player 2 plans to choose r. However, r is an optimal plan for player 2 only if he or she is in the neutral state of mind if the decision node is reached. If player 2 plans r, and $\tau_2 \leq 400\varepsilon$, then player 1 can randomize such that player 2 transitions to the fearful state of mind, and prefers to deviate to c. Assume instead that player 2 plans to choose c. Choosing c is an optimal plan only if player 2 is in the fearful state of mind when his or her decision node is reached. However, the only outcome with a negative material payoff for player 2 is $((a, v), r)$. If player 2 plans c, then his or her peril, initial and updated, is zero. Player 2 remains in the neutral state of mind and prefers to deviate to r. The own-plan dependency of player 2's peril leads to self-negating beliefs. Further, since player 2 has strict preferences over c and r for all values of τ_2, player 2 cannot have an optimal non-degenerate plan. Consequently, if $\tau_2 \leq 400\varepsilon$, then player 2 cannot have an optimal plan and no SE exists.

There are at least two possible solutions to the problem posed by this example. The first is to construct an equilibrium that is consistent with rationality constraints by smoothing the utility function around τ. The utility function u can thus be approximated by a continuous function u' which has a value equivalent to u's value accept in the boundary around τ where u is discontinuous. The second approach is to study ε equilibria (see e.g. Fudenberg & Levine, 1986; Jackson et al., 2012; Monderer & Samet, 1989; Radner, 1980).

There is one additional source of non-existing equilibrium in this model. Consider the case of a player 1 with an incentive to choose the probability of a as *small* as possible *without* causing player 2 to transition to the fearful state of mind. Since player 2's peril is decreasing in the probability of a and player 2 transitions to the fearful state of mind at h if $P_2'(h|\alpha_2) \leq \tau_2$, a player 1 with these incentives does not have an optimal plan and no equilibrium exists.

4 Bank runs

Fear can be of importance also when the players' incentives are aligned. The game studied in this application is a multi-stage game between three personal players and chance. Note that the personal players are the bank clients. The bank is considered passive throughout the game. The game is a simplification of Diamond and Dybvig's (1983) seminal bank run game and is inspired by the experimental work of Garratt & Keister (2009).

The game proceeds in three stages. In stage 0 the players have 1 monetary unit and simultaneously decide whether to deposit it in the bank (d) or keep it in the mattress (k). Chance is passive in stage 0. In stage 1 and 2 of the game, all players with a deposit in the bank decide whether to withdraw it (w) or let it remain in the bank (r). The player's decision tree is illustrated in Figure (4).

If a player decides to withdraw the deposit, then the deposit is withdrawn with probability 1. In addition, the players face an exogenous risk of 'needing money tomorrow': in each stage each player faces the probability $\varepsilon \in (0, 1)$ of being selected by chance and forced to withdraw. I assume that the exogenous risk of withdrawal is independent both between stages and between players.

Let D denote the number of players who deposited their money in the bank in stage 0. After stage 0, the bank's assets equals the units deposited by the players. If $D > 1$, then the bank invests in a technology for an immediate cost of 1. This technology can provide a return after stage 2. The bank needs to liquidate the

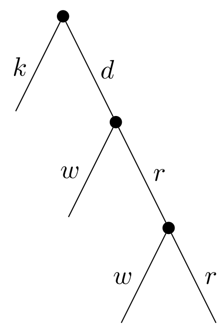

Figure 4: The personal players' decisions in the bank run game.

assets if the number of withdrawals in stage 1 and 2 is weakly greater than $D-1$. Let W_k, $k \in \{1,2\}$ denote the number of withdrawals in stage 1 and 2 of the game. The bank liquidates the assets in stage 1 if $W_1 \geq D-1$, and in stage 2 if $W_1 + W_2 \geq D-1$. If the bank does not liquidate the assets in either stage, then the technology 'bears fruit' and provides a monetary return.

The payoffs are normalized such that only losing the full deposit is a negative outcome. The normalized payoffs are illustrated in Table 1, conditional on all players depositing their money in stage 0. If no player withdraws the deposit, then they each receives a normalized payoff of 7. If only one player withdraws, then he or she receives a normalized payoff of 1 and the two players not withdrawing receive a normalized payoff of 5 each. If two players withdraw, then they each receives a normalized payoff of 1, while the player not withdrawing loses the full deposit and receives a normalized payoff of -1. This can occur either by the two players withdrawing in the same stage or by one of them withdrawing in stage 1 and the other in stage 2. If all three players withdraw their deposit, then their normalized payoffs depend on the order in which they withdrew. If all three players withdraw simultaneous, then they each receive a payoff of 2/3. If one player withdraws in stage 1 and the other two in stage 2, then the player withdrawing in stage 1 receives a normalized payoff of 1 and the other two a normalized payoff of 1/2. The normalized payoff from not depositing the unit is 1.[17]

There is a multiplicity of SE in this game and the focus of this analysis is on a

[17]This application can be generalized to other payoffs. Crucial for the results is that only the outcome in which the player loses his or her full deposit has a negative material payoff. The other values are used to calculate and compare threshold values for ε for different equilibria.

Table 1: Normalized material payoffs given that all players deposit their unit.

# Withdrawals	$\pi_i(r)$	$\pi_i(w)$
0	7	NA
1	5	1
2	-1	1
3	NA	$2/3$
1, 2	NA	$1, 1/2$

'good strategy profile' in which all players plan to deposit their unit in the bank and not to withdraw in either stage.

Definition 3. The strategy profile σ is a *good strategy profile* if $\sigma_i = (d, r, r)$ for all $i \in I$.

Players with standard preferences Consider the game between players with standard preferences. If they use the good strategy profile, then player i's initial expected payoff is[18]

$$\begin{aligned}\mathbb{E}\left[\pi_i \middle| (3, r)\,; \alpha_i\right] =& (2/3)\varepsilon^3 + (1-\varepsilon)\varepsilon^2 + (1-\varepsilon)^2\varepsilon \\ &+ 2(1-\varepsilon)^2\varepsilon\mathbb{E}\left[\pi_i \middle| ((3,1), r)\,; \alpha_i\right] \\ &+ (1-\varepsilon)^3\mathbb{E}\left[\pi_i \middle| ((3,0), r)\,; \alpha_i\right].\end{aligned} \tag{13}$$

When the players use the good strategy profile, they only withdraw their deposit if forced to do so. The first term is the (exogenous) probability that all players withdraw in stage 1 for a normalized material payoff of (2/3). The second term is the probability that two players withdraw. If player i is one of them, then he or she receives a normalized material payoff of 1, otherwise he or she receives a normalized material payoff of -1. The third term is the probability that only player i withdraws in stage 1 for a normalized material payoff of 1. The fourth term is the probability that a player $j \neq i$ withdraws in stage 1 times the expected material payoff from stage 2. Likewise, the fifth term is the probability that no player withdraws in stage 1 times the expected material payoff from stage 2.

At the beginning of stage 2, the players observe the withdrawals in stage 1. If two or more players withdrew, then a bank collapse has occurred and the

[18]With a slight abuse of notation, the history is written as the number of deposits in stage 0 and the number of withdrawals observed after stage 1.

remaining player (if any) loses all of his or her money. If a player $j \neq i$ withdrew, then player i's expected payoff from not withdrawing in stage 2 is

$$\mathbb{E}\left[\pi_i|\left((3,1),r\right);\alpha_i\right] = 5(1-\varepsilon)^2 + (1/2)\varepsilon^2. \tag{14}$$

If no player withdrew, then player i's expected payoff from not withdrawing in stage 2 is

$$\mathbb{E}\left[\pi_i|\left((3,0),r\right);\alpha_i\right] = (2/3)\varepsilon^3 + (1-\varepsilon)\varepsilon^2 + 11(1-\varepsilon)^2\varepsilon + 7(1-\varepsilon)^3. \tag{15}$$

The payoff from deviating and withdrawing the deposit is 1 in either stage. For $\varepsilon < 0.496$ the *good strategy profile* is an SE when the players have standard preferences.

Players with two states of mind Now consider players who may transition to the fearful state of mind. The maximin action in stages 1 and 2 is to withdraw the deposit. Assume the players have identical fear thresholds, τ_i, and that they use the good strategy profile. In this case, peril is due to the exogenous risk of withdrawal.

Player i's initial peril, the probability that the other two players will be forced to withdraw, is

$$P_i\left(h^0|\alpha_i\right) = (1-\varepsilon)\varepsilon^2 + 2(1-\varepsilon)^3\varepsilon^2 + (1-\varepsilon)^4\varepsilon^2. \tag{16}$$

The first term is the probability that the other two players withdraw in stage 1. The second term is the probability that one of the other players withdraws in stage 1 and the other in stage 2. The third term is the probability that both players withdraw in stage 2.

If no player withdrew in stage 1, then the updated peril is

$$P_i\left((3,0)|\alpha_i\right) = (1-\varepsilon)\varepsilon^2. \tag{17}$$

The updated peril corresponds to the probability that both other players are forced to withdraw in stage 2, but not player i.

The updated peril is smaller than the initial peril and $P_i'((3,0)|\alpha_i) = 0$. There is no increase in peril and the players do not transition to the fearful state of mind. If at least 2 players withdrew in stage 1, then the bank has collapsed. Any remaining player transitions to the fearful state of mind if sufficiently sensitive to fear, but he or she has no action left to take and receives a payoff of -1.

The more interesting case is when only one player withdraws in stage 1. The updated peril for the remaining players is

$$P_i\left((3,1)|\alpha_i\right) = (1-\varepsilon)\varepsilon. \tag{18}$$

The updated peril corresponds to the probability that (only) the other player is forced to withdraw in stage 2.

The increase in peril is

$$P_i'((3,1)|\alpha_i) = (1-\varepsilon)\varepsilon(1-(3(1-\varepsilon)^3+1)\varepsilon). \tag{19}$$

Let $\overline{\tau}$ denote the maximum fear threshold for which the players transition to the fearful state of mind conditional on observing one withdrawal in stage 1, $\overline{\tau} = (1-\varepsilon)\varepsilon(1-(3(1-\varepsilon)^3+1)\varepsilon)$. Once fearful, the players prefer to deviate to w. The plan of choosing r in both stages regardless the outcome of stage 1 can therefore not be an optimal plan for such players.

Proposition 3. If the players are sufficiently sensitive to fear, $\tau_i \leq \overline{\tau}$, then the *good strategy profile* cannot be an SE.

Fear sensitive bank run players may experience a welfare loss measured in material payoffs. Moreover, this welfare loss is also imposed on their fear insensitive co-players. The social welfare maximizing strategy profile that fear sensitive players can coordinate on is to not withdraw in stage 1 and withdraw in stage 2 conditional on one player being forced to withdraw in stage 1. This strategy profile is an SE for $\varepsilon \leq 0.422$. In this SE no player is fearful on the equilibrium path. The players coordinate on withdrawing conditional on observing one withdrawal in stage 1, and withdrawing is not a negative outcome. In addition, when the exogenous probability of withdrawal is sufficiently high, the game has a unique equilibrium in which all players chooses k and no money is invested. When the players may transition to the fearful state of mind, the exogenous probability for this equilibrium to be unique is smaller than for players with standard preferences. Since fear sensitive players cannot commit to r conditional on observing another player withdrawing, they are less willing to choose d to begin with.

Fear can spread in a population of players with different fear thresholds. Consider the game between three players, but only i has $\tau_i \leq (1-\varepsilon)\varepsilon(1-(3(1-\varepsilon)^3+1)\varepsilon$. Assume the players use the good strategy profile, but that player $j \neq i$ is forced to withdraw in the first stage. This causes player i to transition to the fearful state of mind. The third player, player k, knows the value of τ_i and correctly anticipates that player i will withdraw in the next stage. Consequently, his or her peril increases further and may reach τ_k, causing player k to transition to the fearful state as well.

Proposition 4. A player with $\tau_i > \overline{\tau}$ may transition to the fearful state of mind after observing one withdrawal in stage 1 if he or she knows that the other remaining player will transition to the fearful state.

5 Public health intervention

As a final application, consider a simple example of a public health authority who wants to inform the public about accurate estimates of the probability or cost of contracting a disease. This example stem from the observation that in many cases beliefs over the probability or cost of a negative outcome are mistaken.[19]

Consider a decision maker, player 1, who contemplates whether to take a vaccine against a disease. Player 1 becomes immune to the disease if he or she takes the vaccine. Otherwise player 1 faces a positive probability of contracting it.

Player 1 holds correct beliefs over the probability of contracting the disease and the cost of vaccination. However, he or she has underestimated the cost of the disease.[20] Recognizing this, the public health authority launches an information campaign to inform player 1 about the true cost. Note that the public health authority is not considered a player in this scenario.

The probability of contracting the disease (if not vaccinated) is denoted by $\varepsilon > 0$, and the cost of vaccination is denoted by v, $0 < v < 1$. Player 1's prior belief over the cost of the disease is denoted by d, and his or her updated belief is denoted by d', where $d' > d > 1$.

[19]Or they may be correct but still possible to influence.

[20]A related situation is the more ethically dubious case when the authority wants to scare people into a certain behavior by overstating the cost.

The payoffs are normalized such that the only negative outcome is to contract the disease. The normalized payoff from not taking the vaccine and not contracting the disease is 1. The normalized payoff from taking the vaccine is $1 - v$. Finally, the normalized expected payoff of contracting the disease is $1 - d$ and $1 - d'$, when player 1 is uninformed and informed, respectively. Player 1's maximin action is to take the vaccine.

First consider a player 1 who initially plans to take the vaccine. Player 1 is risk neutral in the neutral state of mind and takes the vaccine if $v \leq \varepsilon d$. Because the only negative outcome is contracting the disease, player 1's initial peril is zero, $P_1(h^0; \alpha_1) = 0$. He or she will take the vaccine and become immune to the disease. The public health authority informs player 1 about the correct cost of the disease. However, since player 1 plans to take the vaccine, the updated peril is zero. There is no change in peril and, since $v \leq \varepsilon d < \varepsilon d'$, player 1 is unaffected by the information.

Next consider a player 1 who initially plans not to take the vaccine. That is, $v \geq \varepsilon d$. Player 1's initial peril is

$$P_1(h^0; \alpha_1) = \varepsilon \times d. \tag{20}$$

The public health authority informs player 1 about the correct cost of the disease. As in traditional theory, player 1 changes his or her plan of taking the vaccine if $v \leq \varepsilon d'$. In addition, a fear sensitive player may transition to the fearful state of mind if the information causes a sufficiently large increase in peril.

Player 1's increase in peril after receiving the information is

$$P_1'(info; \alpha_1) = \varepsilon(d' - d). \tag{21}$$

Player 1 transitions to the fearful state of mind if the increase in the cost of the disease is sufficiently large such that τ_1 is reached. In the fearful state of mind, player 1 maximizes minimum payoff by taking the vaccine also when $v > \varepsilon d'$. In other words, a fearful player 1 will take the vaccine also when the cost outweigh the benefits.

Proposition 5. Fear sensitivity can strengthen the behavioral response to information that increases the expected cost of negative outcomes.

As in traditional theory, if $v \in (\varepsilon d, \varepsilon d']$, then information alters behavior. A player who remains in the neutral state of mind after receiving the information takes the vaccine also when $\varepsilon d < v \leq \varepsilon d'$. However, a player who transitions to the fearful state of mind after receiving the information takes the vaccine regardless the value of v. Fear sensitive players with a high cost of vaccination may overconsume the vaccine.[21] On the other hand, vaccines often have a positive externality. Consider for example a disease which is highly transmissible, but the players fail to take the positive externality into account. In such a situation, a fear response may increase social welfare.

6 Concluding Remarks

This paper presents a model of players who can transition from a neutral to a fearful state of mind. In the neutral state of mind, the players maximize own expected material payoff. In the fearful state of mind, they maximize own minimum material payoff. The players are in the neutral state of mind when the game begins and transition to the fearful state after a sufficiently large increase in the expected cost of negative outcomes; outcomes bad enough to potentially instill fear when anticipated.

The focus of this paper is on how fear affects behavior through its effect on a player's risk aversion, but fear is also an unpleasant emotion. While the disutility of experiencing fear is an incentive for players to avoid fearful situations, this paper abstract from this to focus on how behavior is affected once a fearful state is reached.

The interactions studied in this paper are assumed to be sufficiently fast such that there is no time to transition back to the neutral state of mind. However, many interactions are slower and allow for fear to fade away. Time can be explicitly modeled using Psychological game theory (Battigalli et al., 2019) and the model can be expanded to interactions which take place over several time periods. The passing of a time period may cause a player to transition back to the neutral state of mind. Such a model requires an assumption regarding a fearful player's degree of sophistication which is not necessarily the same as the degree of sophistication in the neutral state of mind.

[21] The cost of vaccination may for example include expected side effects that may vary between players with different health statuses.

The transition from the neutral to the fearful state of mind is determined by beliefs over outcomes. This definition rules transitions caused by a player's beliefs over others' beliefs. For example, someone might become fearful if he or she believes that their boss believes that he or she has low productivity. However, one can model this situation as a player with the negative outcome of losing his or her job. An increase in the belief that the boss believes that he or she has low productivity causes an increase in the probability that he or she will lose the job, which may trigger fear.

Players' increased concern with risk in the fearful state of mind causes an strengthened change in behavior that may appear as an overreaction. This observation is in line with empirical and experimental research that finds a 'residual' change in behavior after bad news or adverse events (Guerrero et al., 2012; Guiso et al., 2018; Piccoli et al., 2017; Wang & Young, 2020). In other words, the behavioral response is stronger than what can be explained by traditional factors alone. The residual change in behavior is however consistent with an emotion-based change of the utility function.

This paper studies sophisticated players who can perfectly predict own and others' state transitions and the behavioral consequences thereof. In the sequential equilibrium for psychological games, the players are certain about own and others' plans, and never change their minds about them. Any deviations from the plans are interpreted as mistakes. This is a strong assumption, especially since the players' decision utility may depend on both own and others' plans. However, the analysis shows that fear is of importance even when players can perfectly predict own and others' state transitions. Whereas the study of naïve players who cannot perfectly predict own emotional response is outside the scope of this paper, it is an interesting avenue for future research.

It is natural to ask whether fear, modeled as belief-dependent risk aversion, makes sense from an evolutionary perspective. Aumann (2019) argues that people use rules to guide their behavior and that these behavioral rules are the product of evolutionary forces. Rather than maximizing utility over actions, people adopt rules of behavior that do well in usual, naturally occurring situations. A simple example of such a rule related to fear could be 'maximize expected utility when in a safe environment while minimize the worst losses when in peril'.

Besides robberies, bank runs, and public health policy, fear may be important in understanding for example conflict; environmental dangers and climate risk; and financial decision making. These applications are potentially of great importance and are left as an avenue for future research.

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